TYPE I ERROR PROBABILITIES AND POWER OF THE RANK AND PARAMETRIC ANCOVA PROCEDURES

By

SAMUEL L. SEAMAN

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To Aaron Phillip

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Samuel L. Seaman

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The purpose of the present study was to examine the Type I error rates and statistical powers for the parametric and rank analysis of covariance procedures for a variety of experimental conditions that have not been considered in previous research. The probability of obtaining a significant statistic, using the parametric and rank analysis of covariance procedures, was estimated for a number of experimental conditions defined in terms of the conditional distributions for two groups in a randomized experiment. The two conditional distributions were both normal, both skewed to the same degree, but in different directions, or both skewed in the same direction but to different degrees. Two ratios of conditional variance were investigated for each of the three possibilities described above. A ratio of one:one was used to simulate the experimental condition

of homogeneous conditional variances. A ratio of two:one was used to simulate a modest degree of between group heterogeneity of variance. For each type of pair of conditional distributions, both equal and unequal conditional means were studied. In addition, sample size and size of conditional variance were also manipulated.

Results were interpreted in terms of Type I error probabilities and statistical power for testing two different null hypotheses, equality of two conditional means, or equality of two conditional distributions. It was concluded that the parametric analysis of covariance procedure is to be preferred as a test of both hypotheses provided that the conditional distributions are normal, or the conditional distributions are non-normal and the correlations among height of conditional mean function, degree (or direction) of skew, and size of conditional variance are expected to be positive. For other combinations of these correlations, the choice of a statistical test becomes much more complicated, and depends upon the sample size, size of conditional variance, form of conditional distributions, and hypothesis to be tested. In most of the cases studied, however, the choice of a particular statistical test was inconsequential and a general use of the parametric procedure was indicated.

CHAPTER I

INTRODUCTION

The analysis of covariance is often used to analyze data obtained in educational research studies conducted using the pretest-posttest control group design (Campbell and Stanley, 1963). In the analysis, the pretest is treated as the covariate and represents a source of variation that is expected to be correlated with the dependent variable. The posttest is treated as the dependent variable. When used with a randomized experiment, which is the context assumed in this dissertation study, the purpose of analysis of covariance is to increase power and efficiency relative to that obtained using an analysis of variance of the posttest data.

The linear model for a one-way, fixed effects design with one dependent variable and one covariate is given by

$$Y_{ij} = \mu + \alpha_j + \beta_w(X_{ij}) + \epsilon_{ij}$$

where Y_{ij} is the i^{th} observation for the j^{th} treatment on the dependent variable, X_{ij} is the covariate for subject i in treatment level j, μ is the constant for overall mean response conditional upon X, α_j is the constant for the

j^{th} treatment effect, β_w is the linear regression coefficient of Y on X, and ϵ_{ij} are random variables assumed to be normally and independently distributed with mean zero and common variance.

The assumptions for analysis of covariance are well known (see for example, Huitema, 1980); however, they are presented here for completeness:

a) The regression slopes associated with the various treatment groups are equal.

b) The relationship between the covariate and dependent variable is linear.

c) Within each group the distribution of dependent variable scores conditional on the covariate is normal.

d) For each value of the covariate measure, the variances conditional on the covariate are homogeneous both within and between treatment groups.

The latter two assumptions, conditional normality and homogeneity of variance, are most pertinent to the following discussion.

These four assumptions must be met for the F ratio, calculated in the analysis of covariance procedure, to be distributed as the F distribution. In practice, it is unlikely that all of the assumptions of the analysis are met exactly. If one or more of the assumptions are violated, the researcher may decide to proceed with the

analysis at some risk, or abandon the model and use instead an alternative procedure that does not require all of the assumptions of the parametric analysis of covariance model. Several nonparametric analysis of covariance strategies have been reported in the literature (Quade, 1967; Puri and Sen, 1969; McSweeney and Porter, 1971; Burnett and Barr, 1977; Shirley, 1981). These procedures do not require assumptions b, c or d given above.

If the researcher does not have a way of determining what effect violations of the assumptions might have on his analysis, then interpretation of the results may be inaccurate (Zikri, 1983). It is not surprising then, that a number of research studies have considered the robustness properties and statistical power of the parametric analysis of covariance model when assumptions have been violated. (Atiqullah, 1964; Levy, 1980; McClaren, 1972; Peckham, 1970; Rogosa, 1980; Shields, 1978; Thomson, 1980; Wildemann, 1974). Additional studies have compared these same properties for the parametric analysis of covariance and one or more of the nonparametric alternatives (Abunnaja, 1983; Conover and Iman, 1982; McSweeney and Porter, 1971; Olejnik and Algina, 1983, 1984; Olejnik, Algina and Abdel-Fattah, 1984). In the present study, the type I error rates and statistical powers of the parametric and rank analysis of covariance procedures were examined

under violations of the conditional normality and/or homogeneity of variance assumptions.

The Problem

Huitema (1980) in summarizing much of the research in the area reports that, when sample sizes are equal, parametric analysis of covariance may not be drastically affected by violations of the assumptions of conditional normality, and between or within group homogeneity of conditional variances. If, however, the assumptions of normality and/or between group homogeneity of conditional variances, are seriously violated, especially when sample sizes are unequal, the parametric analysis of covariance procedure may yield biased F ratios.

Olejnik and Algina (1984) reviewed the alternative nonparametric analysis of covariance procedures. They provided a detailed discussion of each procedure, and comparative information relative to type I error rates and statistical power. Olejnik and Algina found that, when the parametric analysis of covariance is robust to simulated violations of the assumptions of conditional normality and between and within group homogeneity of variance, the nonparametric alternatives suggested by Quade, Puri and Sen, and McSweeney and Porter also have acceptable type I errors. (Their review also suggests that the Burnett and Barr, and Shirley procedures frequently yield unacceptable type I errors and should not be considered viable alternatives to parametric analysis of

covariance). Moreover, even when extreme non-normality results in discrepancies between actual and nominal type I error rates for the parametric analysis of covariance, the Quade, Porter and McSweeney and Puri and Sen procedures tend to yield appropriate type I error rates. Olejnik and Algina did not review any investigations of violations of between group homogeneity either alone or in combination with conditional non-normality, and the literature review conducted for this dissertation did not uncover any such studies.

A number of researchers have concluded that when distributional assumptions are violated, the nonparametric procedures should be used in place of their parametric counterparts even when the parametric procedures are robust to violations of those assumptions. Blair and Higgins (1980), while discussing the Wilcoxin test as an alternative to the parametric t-test, make this recommendation. They cited the tendency for the nonparametric procedure to be more powerful than the parametric procedure for a wide variety of non-normal distributions. And, McSweeney and Porter (1971), in discussing their nonparametric analysis of covariance offer this advice:

> The fact that the relative advantage of the parametric tests was slight even when the assumptions necessary for their valid use were completely satisfied suggests that little loss, and possible considerable gain, in power will result from the more general use of these nonparametric analouges. (p. 39)

McSweeney and Porter's suggestion that their nonparametric approach might prove more powerful for a variety of non-normal conditional distributions has since been substantiated by Abunnaja (1983), Conover and Iman (1982), Olejnik and Algina (1983, 1984), and Olejnik, Algina and Abdel-Fattah (1984).

A common characteristic of all the investigations comparing the nonparametric alternatives to the parametric analysis of covariance is that each considered situations in which there were no between group differences in the conditional distributions. The only between group differences examined were differences in the conditional means. Thus their results do not provide guidance for the researcher who must choose between parametric and nonparametric analysis of covariance procedures in situations where between group differences in skew and/or variance exists alone or in conjunction with between group differences in conditional means.

The summary of the literature presented above suggests a void concerning the comparison of parametric and nonparametric analysis of covariance. In the first place, nothing is known about how nonparametric alternatives to analysis of covariance function when there is between group heterogeneity of variance. Secondly, nothing is known about how the parametric analysis of covariance or the nonparametric alternatives function when the conditional distributions differ, between groups, in skew.

The general purpose of this study was to examine properties of the parametric analysis of covariance and a nonparametric alternative for the experimental conditions characterized above. It was felt that results of the present study would then provide the necessary information to fill this void. The rank analysis of covariance was chosen as the nonparametric analog in this study since it is computationaly more convenient than and provides power estimates similar to Quade's and Puri and Sen's procedures. These two procedures are the only other nonparametric analysis of covariance procedures that tend to yield appropriate Type I errors (Olejnik and Algina, 1984). In the rank transform procedure the dependent variable and covariate scores are transformed by ranking the original scores without regard to treatment groups. The transformed data are then analyzed using the usual parametric analysis. The test statistic is referred to the F distribution with the same degrees of freedom as the parametric case.

Purpose of the Study

Most research designs for comparing the effectiveness of educational treatments yield data which can be analyzed by both parametric and nonparametric tests. In making a choice between the parametric analysis of covariance procedure and the nonparametric rank analysis of covariance procedure it would be useful to have estimates of the actual probabilities of type I error and the actual powers of the

two tests for a variety of experimental conditions where the assumptions of conditional normality and/or homogeneity of variance have been violated. Given this information and a knowledge of the distributions encountered in their studies, researchers can choose the most powerful test from those that are known to yield actual type I error rates near the nominal alpha level. Such tests are said to preserve their level.

Recently there have been a number of investigations (Abunnaja, 1983; Conover and Iman, 1982; Olejnik and Algina, 1983, 1984; Olejnik, Algina and Abdel-Fattah, 1984) of parametric and nonparametric analysis of covariance which provided information of the kind described above. The nonparametric analysis of covariance procedure employed in these studies was the rank analysis of covariance procedure suggested by McSweeney and Porter (1971). Abunnaja (1983) and Olejnik and Algina (1984) also examined the nonparametric alternatives suggested by Burnett and Barr (1977), Puri and Sen (1969) and Quade (1967). A comprehensive review of these studies will be provided in the next chapter, but of interest now are the general findings. Taken together, the results of these studies suggest that for most experimental conditions, both the parametric and rank analysis of covariance procedures tend to preserve their levels for a wide variety of non-normal conditional distributions. In terms of power, however, the parametric procedure enjoys

significant power advantages in certain situations while the rank procedure provides a more powerful test in other situations.

While the Abunnaja, Conover and Iman, and Olejnik, Algina and Abdel-Fattah studies provide useful information for the researcher who must choose between the parametric and rank of covariance procedures, they do not address two important situations which have become the logical motivation for this study. A first consideration was to examine the type I error rates and powers of parametric analysis of covariance and rank analysis of covariance when they are used to test the hypothesis of equality of conditional means, and the conditional distributions for the groups differ in scale and/or shape. Here, preservation of the level of the parametric analysis of covariance is in question because the conditional distributions are non-normal, and robustness to this kind of non-normality has not been examined by previous research in the field. Preservation of the level of the rank analysis of covariance procedure is questionable since the researcher is interested only in detecting differences in conditional means, and the rank analysis of covariance procedure may be sensitive to between group differences in shape, scale, or shape and scale (Savage, 1962).

A second purpose was to examine statistical powers of the parametric and rank procedures when used to test the hypothesis of equality of conditional distributions, and

those conditional distributions may differ in location and scale, location and shape, or location, scale, and shape. With regard to preservation of level in this case, the work of Abunnaja, Conover and Iman, and Olejnik and Algina has shown that if the joint covariate-dependent variable distributions are the same for both groups, then over a wide range of non-normal conditional distributions both the rank and parametric analysis of covariance procedures tend to yield actual alpha levels near the nominal alpha levels. (An exception did occur in the Conover and Iman (1982) study where the parametric analysis of covariance procedure proved to be somewhat conservative with extremely non-normal conditional distributions.) In this case then, the general concern of this study was to examine the comparative powers of the rank and parametric analysis of covariance procedures when they are used to test the hypothesis of equality of conditional distributions, and it is expected that those distributions differ in location and shape, location and scale, or location, shape and scale.

The two cases described above were investigated using Monte Carlo simulation procedures. The scenario was a randomized experiment comparing two experimental groups. For all experimental conditions simulated, the scale and shape of the conditional distributions were independent of the covariate scores and a covariate by treatment interaction did not exist. As a result, conclusions about the utility

of the parametric and rank analysis of covariance procedures for testing hypotheses about conditional means and conditional distributions also apply to testing the corresponding hypotheses about marginal means and marginal dependent variable distributions.

Significance of the Study

Campbell and Stanley (1963) express great concern for validity in experimentation:

> Internal validity is the basic minimum without which any experiment is uninterpretable: Did in fact the experimental treatments make a difference in this specific experimental instance? (pg. 5)

The analysis of covariance is often used in educational research to answer this question. Kirk (1982) identifies the advantages of using the analysis of covariance procedure:

a) There is a reduction in experimental error and, hence, an increase in statistical power.

b) Bias caused by differences among experimental units is reduced when those differences are not attributable to the manipulation of the independent variable.

The advantages given above may not exist when assumptions underlying the model have not been met. If one or more violations are suspected, a nonparametric alternative may be the more appropriate choice. The validity of the conclusion obtained using either procedure, parametric or nonparametric, will certainly be affected by the researchers'

knowledge of the effect that the violations had on the final results of their analyses. Studies examining the type I error rates and statistical powers of the parametric and nonparametric analysis of covariance procedures have provided researchers with empirical evidence useful in making this choice for a number of experimental conditions. The present study examines similar properties of the two procedures for additional experimental conditions where the assumptions of conditional normality and/or between group homgeneity of variance have been violated in a distinctive way. Results of the study will provide additional guidelines that should prove useful in making an astute choice between the parametric and nonparametric statistical procedures, thereby enhancing the probability of accurate and valid conclusions in educational experimentation.

CHAPTER II

REVIEW OF RELATED LITERATURE

Overview

In a published review of the analysis of variance and analysis of covariance procedures, Glass, Peckham, and Sanders (1972) summarized the research that had examined the effects of violations of the assumptions of fixed effects analysis of variance and covariance. While much of their article dealt with the analysis of variance procedure, a number of studies that had investigated the analysis of covariance procedure were also reviewed. Some of the earlier studies cited, suggested that the analysis of covariance was robust to violations of the assumptions of normality and homogeneity of variance (Cochran, 1957; Winer, 1962). The analysis of covariance was shown, analytically, to be insensitive to violations of the normality assumption when the covariate is approximately normally distributed (Box and Anderson, 1962; Atiquallah, 1964). Potthoff (1965) suggested that the sensitivity of the analysis of covariance to heterogeneity of conditional variances depended on the ratio,

$$n_1\sigma_{x1} \; / \; n_2\sigma_{x2} \; .$$

This would suggest that the sensitivity of the analysis of covariance to heterogeneity of conditional variances depends upon the covariate, X, and sample size as well. Peckham (1968) concluded that inequality of regression slopes, unless extreme, had little effect on the analysis of covariance procedure. Atiquallah (1964), however, had suggested that the analysis of covariance was insensitive to unequal regression slopes only if the means of the concomitant variables are equal or the within group sums of squares are equal. Finally, based on several investigations examining unreliable covariate measures (Cochran, 1968; Lord, 1960; Porter, 1967), Glass et al. conclude that,

> as errors of measurement in X increase, the analysis becomes more like the corresponding analysis of variance. Thus, (1) the increase in precision afforded by ANCOVA is attenuated and (2) there is less reason to assume that "the groups have been statistically equated" on the concomitant variable. (pg. 281)

In the wake of Glass, Peckham, and Sanders' (1972) article, numerous investigations of the effects of violation of the assumptions of covariance analysis have been conducted. Hamilton (1976, 1977) was particularly interested in the assumption of homogeneity of regression slopes and a previous study that had been performed by Peckham (1968). In particular, Hamilton (1977) was interested in the effect that unequal regression slopes might have not only on type I errors, but also

on the power of covariance analysis. Hamilton found that the analysis of covariance was apparently robust to violations of homogeneity of regression when group sizes were equal. When group sizes were unequal, and the homogeneity of regression assumption was violated, large discrepancies between actual alpha levels and corresponding nominal alpha levels were observed. In terms of power, when sample sizes were equal, violation of homogeneity of regression had little effect on the statistical power. In another empirical investigation, Levy (1980) examined the effects of unequal regression slopes and/or conditional non-normality. Levy's results suggest that the analysis of covariance procedure is robust to violations of the homogeneity of regression slopes and/or conditional normality when sample sizes are equal. The analysis of covariance procedure also appeared to be robust to violations of conditional normality, whether or not sample sizes were equal, so long as the regression slopes are equal. When assumptions of conditional normality and homogeneity of variance were violated simultaneously, and sample sizes were unequal, the actual alpha levels for the analysis of covariance procedure began to deviate significantly from the nominal alpha levels.

A number of recent analytical studies investigating the effect of heterogeneity of regression slopes

(Hollingsworth, 1980; Rogosa, 1980) provide contradictory conclusions but in general suggest that unequal slopes produce biased test statistics. In addition, Hollingsworth (1980) concluded that statistical power is also affected by heterogeneity of regression slopes. Hsu (1983) is quick to point out however, that practical significance of the bias produced by assumption violations cannot be determined analytically, and suggests that despite the obvious problems associated with Monte Carlo studies, they should in fact be carried out to supplement analytical studies. Finally, Abunnaja (1983) examined the effects of violation of conditional normality and/or homogeneity of regression, again with equal and unequal sample sizes. Abunnaja extended the Levy (1980) study by considering, as well, several nonparametric alternatives to the parametric analysis of covariance. The results of Abunnaja's study are consistent with those of Levy, suggesting that the parametric analysis of covariance is robust to the violations singly and in combination, so long as sample sizes are equal and the differences in regression slopes are not extreme. The nonparametric procedures were also found to preserve their levels for the violations examined in the study.

Earlier in this discussion it was suggested that the assumptions most pertinent to the present study were the assumptions of conditional normality and

homogeneity of conditional variances. A number of studies have examined the effects of violating one or both of these assumptions, and are discussed in detail below.

Wildemann (1974) conducted a Monte Carlo study in which she investigated the effects on the covariance analysis of a conditional non-normality resulting from the use of measuring instruments that yield only discrete observations on a limited score scale. Her results suggest that covariance analysis preserves its level for varying degrees of skewness or discreteness in the covariate and/or dependent variable. Another Monte Carlo investigation conducted by Shields (1978) examined the effect of violating the assumptions of homogeneity of conditional variances on the analysis of covariance and the Johnson-Neyman technique. The homogeneity of variance assumption may be violated in two ways (Elashoff, 1969). The first form of violation occurs when the variance of the dependent variable scores is constant across treatment groups for all values of the covariate, but there is a distinct relationship between the covariate and the variance of the dependent variable (within group heteroscedasticity). The second form of this violation occurs when the group variances on the dependent variable are different, but within each group, the variances of the dependent variable are constant for all values of the covariate (between group heteroscedasticity).

In her study Shields simulated both forms of the violation and investigated the effects such violations had on type I error rates. Her results suggest that the analysis of covariance is robust to the violations of assumptions of within group homoscedasticity (form 1) and/or between group homogeneity (form 2) of variance, provided sample sizes are equal. When sample sizes differed, violation of the assumption of within group homoscedasticity of conditional variances had little effect on type I errors. For different sample sizes a violation of the between group homoscedasticity of variance assumption, however, resulted in unacceptable actual alpha levels when compared to the nominal alpha levels. When sample size and variance were positively correlated, the analysis provided a conservative hypothesis test. For a negative correlation between sample size and variance, the hypothesis test was liberal. Shields did not consider the effects that heteroscedasticity and/or heterogeneity had on the power of the statistical procedures.

In a similar study, Zikri (1983) examined the effect of violation of the assumptions of between group homogeneity of conditional variances and/or conditional normality. The results of Zikri's study parallel those of Shields. When sample sizes were equal, the analysis of covariance procedure was robust to violation of the conditional normality and/or homogeneity of variance assumptions. When sample sizes were unequal, the same test biases were

found as in Shields' study. Zikri also considered the effects violations of the above assumptions had on statistical power, when actual alpha levels were found to be appropriate. When sample sizes were equal and covariate means were equal, violation of the conditional normality and/or homogeneity assumptions had no real affect on statistical power of the analysis of covariance procedure. If sample sizes differed, or if the covariate means were unequal, the power of the statistical procedure was found to be greatly reduced for a large number of the simulated conditions.

Thomson (1980) conducted an empirical investigation into the effect of heterogeneity of within-cell population regression coefficients and heterogeneity of within-cell population error variances, on type I error rates of the analysis of covariance procedure. Again, when sample sizes were equal, the violations had little effect on type I errors. If sample sizes are unequal, the violation of homogeneous variances results in aberrant actual alpha levels when compared to the nominal alpha levels.

Parametric vs. Nonparametric ANCOVA

As indicated earlier, the general purpose of the present study was to provide guidelines which might assist the researcher who must choose between the parametric analysis of covariance or an alternative nonparametric alternative. Studies by Abunnaja (1983) and Olejnik and

Algina (1984) were cited earlier. The general findings of these studies suggested that of the common nonparametric alternatives, those offered by McSweeney and Porter, Puri and Sen, and Quade were found to preserve their levels for a wide range of non-normal conditional distributions. It was further noted that the procedure suggested by McSweeney and Porter was computationally the easiest procedure to implement for a given data set, and had power estimates quite similar to the other acceptable procedures. The focus of the following literature review is therefore on studies which have compared the parametric analysis of covariance procedure and at least one of its nonparametric analogs.

A study by Abunnaja (1983) cited earlier in a discussion of the effect of violation of the homogeneity of regression assumption, also considered the effects of conditional non-normality and the combined effects of conditional non-normality and heterogeneity of regression, for the parametric analysis of covariance and a number of nonparametric alternatives. The results of the study suggested that both the parametric and three of the nonparametric procedures had acceptable type I errors for conditions where conditional normality and/or homogeneity of regression assumptions had been violated. The three nonparametric procedures were those suggested by McSweeney and Porter, Puri and Sen, and Quade.

Further, the results suggested that the three non-parametric procedures cited above had comparable empirical power, but their power values were consistently smaller than powers of the parametric procedures examined. Conover and Iman (1982) compared the parametric analysis of covariance and rank analysis of covariance procedures by simulating data for the situation in which four treatments were to be compared. The conditional distribution for each group was normal, lognormal, exponential, uniform or Cauchy. In the null case the joint covariate-dependent variable distributions were the same for all four groups. The level of the rank procedure was preserved for all five distributions. The level of the parametric procedure was preserved with the normal, exponential and uniform distributions, but was conservative with the lognormal and Cauchy distributions. In the non-null situation the covariate distributions were the same for the four groups but the conditional distributions differed in location for five of the six pairs of groups. The parametric ANCOVA was more powerful with the normal and uniform distributions but less powerful with the exponential, lognormal, and Cauchy distributions. Olejnik and Algina (1983) conducted a simulation which focused on the two-group situation. Like Conover and Iman they studied the effects of non-normal conditional distributions. However, the degree of

conditional non-normality was much less extreme than in the Conover-Iman study. In addition, Olejnik and Algina investigated the effect of within group heteroscedasticity and the combined effects of conditional non-normality and within group heteroscedasticity. In the null case the joint covariate-dependent variable distributions were the same for both groups, while in the non-null case only the conditional means differed across groups. Olejnik and Algina found that the level of both the rank and parametric ANCOVA was preserved with non-normal, heteroscedastic, and non-normal and heteroscedastic conditional distributions. They also found that when the conditional distributions differed in location the rank procedure can have a small advantage if the conditional distributions are leptokurtic and the covariate-dependent variable correlation is small ($\rho = 3$ in their study). In addition the rank ANCOVA can have a substantial power advantage if the conditional distributions are leptokurtic, skewed and heteroscedastic, skewed and leptokurtic, or skewed, leptokurtic and heteroscedastic. The parametric ANCOVA can have a small power advantage if the conditional distribution is skewed and the covariate-dependent variable correlation is large ($\rho = .7$ in their study).

Olejnik, Algina and Abdel-Fattah (1984) examined the effects that violations of conditional normality and/or within group homogeneity of variance had on the statistical

power of the parametric analysis of covariance procedure and on the differences in statistical power between the parametric and rank analysis of covariance procedures. Results of their study suggest that the statistical power of the parametric analysis of covariance procedure is little affected by modest violations of the conditional normality and/or within group homogeneity of variance assumptions. The statistical power of the rank analysis of covariance procedure, however, was found to increase with the introduction of conditional non-normality and/or within group heterogeneity of variance. The greatest increases were observed for conditions where the relationship between the covariate and posttest measures was relatively weak. Olejnik, Algina and Abdel-Fattah also reported that the strength of the covariate-dependent variable relationship had a significant effect on the statistical powers of the two procedures (parametric and rank ANCOVA). They found that when the covariate-dependent variable relationship was moderate or stronger (ρ=.7), there was little reason to use the rank analysis of covariance procedure, as power advantages, when they did exist, were minimal. When the covariate-dependent variable relationship was weak (ρ=.3), however, the rank analysis of covariance procedure did enjoy power advantages over the parametric analysis of covariance. Olejnik, Algina and Abdel-Fattah also developed power curves, and examined the power differences over the entire range of the power curves. They found that the power differences between the parametric and rank analysis of covariance

procedures were not consistent over the entire range of each power curve. The largest differences in power occurred for the most part, at the middle of the power curve (parametric powers ranging from .4 to .6). A general conclusion derived in their study was that power differences (parametric vs. rank analysis of covariance) were a function of the strength of the covariate-dependent variable relationship, the degree to which assumptions were violated, and the statistical power of the parametric analysis of covariance.

Summary

The research findings reviewed above suggest that when sample sizes are equal, the analysis of covariance procedure tends to be robust to moderate violations of a number of the usual assumptions both singly and in combination. In those studies where a nonparametric analysis of covariance procedure was examined, the nonparametric procedure too, yielded actual alpha levels near the nominal alpha for a variety of violations of assumptions. When unequal sample sizes were encountered, however, the parametric procedure was found to yield actual alpha levels that were significantly different from the nominal alpha levels.

In terms of statistical power, research findings reviewed provide a number of interesting conclusions. When violations of the conditional normality assumption are extreme, the rank analysis of covariance usually enjoys significant power advantages over the parametric analysis of covariance procedure. For less extreme violations of the conditional normality assumption, smaller power advantages associated

with the rank analysis of covariance were found when the covariate-dependent variable correlation was weak. If the covariate-dependent variable relationship is large the parametric procedures tend to enjoy small power advantages over the rank procedure. Finally, considerable power advantages favoring the rank ANCOVA were observed for the combined violations of the conditional normality and homogeneity of within group variance assumptions. These differences were found to increase with any associated increase in sample size. However, the power advantages were shown to be greatest for the central portion of the power curves.

A common characteristic of all investigations comparing the nonparametric and parametric procedures was that each study considered situations where no between group differences in conditional distributions existed. The only between group differences, were differences in conditional means. The results of these studies, therefore, do not provide guidance for the researcher who must choose between the parametric and nonparametric analysis of covariance procedures in situations where between group difference in skew and/or variance exist alone or in conjunction with between group differences in conditional means.

The general purpose of this study then was to examine properties of the parametric analysis of covariance and a nonparametric analog for the unique experimental conditions characterized above. Results of the present study should

then provide the necessary information to fill the void that now exists, enabling the researcher to make an astute choice of a statistical procedure in situations where between group differences in skew and/or variance are expected.

CHAPTER III

METHODOLOGY

In the present study, data for a one-way fixed effects analysis with two treatment groups, one dependent variable, and one covariate, were simulated. Four factors were of interest: a) sample size, b) size of the conditional variance, c) form of the conditional distribution, and d) heterogeneity of conditional variances. The levels of these factors were combined so that a number of experimental conditions could be simulated. The specific levels of the factors examined, the method of data generation, and combinations of all levels investigated are described in detail below.

Design

Sample Size. The study involved comparisons of two groups having equal sizes of either 20 or 40 observations in each. A cell frequency of 20 was seen as being representative of a small experiment, with smaller cell sizes occurring infrequently in educational research. A cell frequency of 40 was chosen as being representative of moderate sized experiments, and more likely to occur in educational studies than would two group experiments with

larger cell frequencies. Numerous research studies have documented the detrimental effects of conducting experiments where sample sizes were unequal, the general consensus being that practitioners should avoid unequal sample sizes. The effect of inequality of sample sizes was therefore not considered in the present study.

Size of the Conditional Variance. Two levels of conditional variance were investigated, $1 - .3^2$ and $1 - .7^2$. In the homoscedastic case these conditional variances resulted in covariate-dependent variable correlations of .7 and .3 respectively. A correlation of .3 represents a relatively weak covariate-dependent variable relationship, while the correlation of .7 represents a relatively strong relationship. Correlations encountered in behavioral science research often occur in the interval from .2 to .7, and hence the correlations above reflect realistic values for this parameter. In the heteroscedastic case the average conditional variance was either $1 - .3^2$ or $1 - .7^2$. This assured that any effect of heteroscedasticity was not confounded with the size of the error term.

Form of the Conditional Distributions. The conditional distributions for the two treatment groups were either both normal or both skewed. For the conditions involving two skewed distributions, there were two distinct cases

to be considered. In one case, hereafter referred to as Case A, the degree of skewness was the same in both groups, but positive for one and negative for the other. For Case A, two levels of skewness were investigated; $\pm$ .25, and $\pm$ .75. Relative frequency distributions based on a random sample of 10,000 observations from distributions with skewness .25 and .75 respectively are presented in Table 1. The levels of skewness were chosen to reflect very small to moderate departures from normality. It was felt that if in practice conditional distributions differ in sign of skew, the degree of skew would be fairly small for behavioral data.

A second case, hereafter referred to as Case B, considered two groups having conditional distributions that were skewed in the same direction, but had degrees of skewness that were different. Here, two different combinations of skew were considered, .25/.50, and .25/.75. A relative frequency distribution based on a random sample of 10,000 observations from a distribution with skewness .50 is also presented in Table 1. The first combination represents groups having mildly skewed conditional distributions that differ only slightly in degree of skewness. The second combination represents groups having slightly skewed and moderately skewed distributions that now differ significantly in degree of skew. Again, it was felt that these combinations

Table 1

Relative Frequency Distribution for Observations of Three Skewed Random Variables

Standard Deviations[a] from the Mean	Degree of Skew .25	.50	.75
$-\infty$,-3.0			
-3.0,-2.0	1.52		
-2.0,-1.0	14.34	17.05	18.20
-1.0, 0.0	36.16	36.91	37.82
0.0, 1.0	31.75	29.57	26.58
1.0, 2.0	13.40	13.25	13.34
2.0, 3.0	2.57	2.87	3.67
3.0, ∞	.26	.35	.39

a. Each frequency distribution is based on 10,000 observations.

would reflect conditions that are most likely to occur in practice.

Ratio of Conditional Variances. Two ratios of conditional variances were considered. The first, a ratio of 1, simulates the experimental condition of homogeneous conditional variances. The second, a ratio of 2, simulates the experimental condition where a modest degree of heteroscedasticity exists. Shields (1978) examined the effect of more severe degrees of heteroscedasticity on the robustness of the parametric analysis of covariance. Again, in the present study experimental conditions likely to occur in actual practice were of primary concern, and hence it was more important to examine the effects of a more modest degree of heteroscedasticity.

Generation of the Data. The equation used to generate the dependent variable data is given by

$$Y = \rho_j \quad X + E\ (S_j)\sqrt{1 - \rho_j^2} + C_j. \qquad (2)$$

For experimental conditions where the conditional mean functions were the same for both groups an initial data set was generated by allowing C_j to equal the constant 0 in both groups. This initial data set was then used to generate data for the non-null case by adding an appropriate constant, C_2, to each of the scores in group 2. Hence, C_2 was set equal to .2, .4, .6, or .8 when the sample size was 20 in each group (.15, .30, .45, or .60 when the sample size was 40 in each group), so that the statistical powers could be compared for various effect sizes. In all

experimental conditions simulated, X was distributed normally with mean 50 and variance 1. To simulate those experimental conditions where conditional variances were equal, S_j was set equal to 1 for both groups. When conditional variances differed for the two groups, one of the S_j was set equal to 1 while the other S_j was set equal to $\sqrt{2}$. Finally, for the homoscedastic case ρ_j was set equal to .3 or .7 simulating the two levels of conditional variance discussed previously. For the heteroscedastic case ρ_j was set equal to .6271629 or .8124038 simulating the two levels of average conditional variance.

The normally distributed covariate data were simulated by generating normal random variables using the Statistical Analysis System (1982) normal function, and adding the constant 50 to each. Again, normal conditional distributions were simulated by generating normal random variables and assigning these values to E in (2). The non-normal conditional distributions were simulated by transforming a standard normal variable (z) to a new variable with mean zero, variance one, and a known degree of skewness. The degree of skewness was manipulated by choosing the appropritate coefficients in the formula $E = c + bz + cz^2 + dz^3$ given by Fleishman (1978).

For each of the experimental conditions studied, observations were simulated from two populations. The simulations were replicated 1000 times for each experimental condition, and the resulting data were analyzed using the analysis of covariance and the rank analysis of covariance procedures.

Conditional Combinations

Conditional Normality. When the conditional distributions were homoscedastic, there were four possible combinations of sample size and the size of conditional variance, and data were simulated for each of these possible combinations. When the conditional distributions were heteroscedastic, it was possible to have either a positive or negative correlation between C_j and S_j. Each of these correlations could have been combined with the four combinations of sample size and size of the conditional variance, for a total of eight additional conditions. However, data simulating the positive correlation can be converted to data simulating the negative correlation by the transformation $Y^* = -Y$. Obviously, the analysis of Y^* and Y will yield the same results and it was unnecessary to simulate both patterns of correlations.

Case A. For the heteroscedastic conditional distributions, there were eight possible patterns of positive and negative correlations among C_j, S_j, and sign of skew. However, since the data were simulated for only two groups, once the correlations between two pairs of these variables were established, the third correlation was automatically determined. In this present study, then, there were really only four distinct patterns of correlations among the three variables. Moreover, if the sign of skew and C_j had a positive correlation, then the two possible patterns of correlations of these variables with S_j involved variables

related by the function $Y^* = -Y$. Similarly if sign of skew and C_j had a negative correlation then the two possible patterns of correlations of these variables with S_j again involved variables related by the function $Y^* = -Y$. As a result only the correlation between sign of skew and C_j was critical, and only one of the two possible conditions involving a positive correlation between these two variables was simulated. Similarly for the negative correlation, between sign of skew and C_j, only one of the two possible conditions was simulated. Table 2 summarizes the experimental conditions simulated for conditional normality and experimental conditions of Case A.

After simulating all experimental conditions for the sample size of 20 in each group, a number of conditions were simulated for the sample size of 40. It became quite obvious that the trends found in simulations for samples of size 20 were, with few exceptions, occurring in similar fashion for identical conditions with the larger sample size of 40 in each group. Due to cost considerations, a limited number of experimental conditions involving a sample size of 40 were chosen.

Case B. Here again, there were eight possible patterns of positive and negative correlations among C_j, S_j, and degree of skew. Data were simulated for only two groups. Therefore, given the correlations between two pairs of these variables, the third correlation was automatically determined, and there were again only four distinct patterns

Table 2

Summary of Conditions Investigated for Conditional Normality and Case A

Correlation[a]	Variance Ratio[b]	Shape of Distribution[b]	Size of the Conditional Variance[c] $1 - .3^2$ Sample Size 20,20	$1 - .3^2$ 40,40	$1 - .7^2$ 20,20	$1 - .7^2$ 40,40
d	1:1	normal	*	*	*	*
	2:1	normal	*	*	*	*
+	1:1	+ .25	*	*	*	*
		∓ .75	*	*	*	*
	2:1	∓ .25	*		*	*
		∓ .75	*		*	*
-	1:1	∓ .25	*		*	
		∓ .75	*		*	
	2:1	∓ .25	*		*	
		∓ .75	*		*	

a. refers to the correlation between sign of skew and height of conditional mean function.
b. refers to the conditional distribution.
c. average conditional variance in the heteroscedastic cases.
d. correlation not defined in these cases.

of correlations among the three variables. The relationship used to limit the number of simulations in Case A, $Y^* = -Y$, was not applicable for the experimental conditions in Case B. As a result, all four patterns of correlations among C_j, S_j, and degree of skew were simulated. Table 3 provides a summary of the experimental conditions simulated in Case B. Again, all experimental conditions were simulated with a sample size of 20 in each group and selected conditions with a sample size of 40 in each group were simulated. Results of these simulations were similar to those for the sample size of 20 and due to cost considerations the simulation of all experimental conditions for the sample size of 40 was not completed.

Summary.

A total of 64 experimental conditions involving non-normal conditional distributions was possible for the factors described above. However, all possible experimental conditions were not simulated for all conditions where sample size was 40 in each group. For each of the conditions chosen from among the possible combinations, data were simulated both for $C_2 - C_1$ equal to zero and for the four non-zero levels of $C_2 - C_1$. When $C_2 - C_1$ was equal to zero, the correlation between C_j and direction or degree of skew was not defined. Therefore, for the non-normal conditional distributions, when $C_2 - C_1$, was equal to zero, the simulation really provided just two replicates of each condition.

Table 3

Summary of Conditions Investigated for Case B

				Size of Conditional Variance[a]			
				$1 - .3^2$		$1 - .7^2$	
				Sample Size			
Group	Variance[b]	Skew[c]	Conditional Mean[d]	20,20	40,40	20,20	40,40
1		+	+	*	*	*	*
2		-	-				
1		+	-	*	*	*	*
2		-	+				
1	+	+	+	*		*	*
2	-	-	-				
1	-	+	+	*		*	*
2	+	-	+				
1	+	-	+	*		*	*
2	-	+	-				
1	-	-	+	*		*	*
2	+	+	-				

Note: The conditions indicated were investigated for both combinations of skew, (.25, .50) and (.25, .75)

a. refers to size of the average conditional variance for heteroscedastic cases.

b. a blank for both groups indicated homoscedasticity. A + indicates the group with the larger conditional variance.

c. A + indicates the group with the more skewed distribution.

d. A + indicates the group with the larger conditional mean function.

CHAPTER IV

RESULTS AND DISCUSSION

Type I Error Probabilities

One purpose of this study was to examine the estimated actual alpha levels of the parametric and rank analysis of covariance procedures, when those procedures were used to test the null hypothesis of equality of conditional means and the conditional distributions encountered differed in scale and/or shape. For each of the experimental conditions studied, observations were simulated from two populations that had equal conditional means. The simulations were replicated 1000 times for each experimental condition, and the resulting data were analyzed using the usual analysis of covariance procedure and the rank analysis of covariance procedure (McSweeney and Porter, 1971) described in an earlier chapter. For completeness, nominal alpha levels of .01 and .05 were used in each case. For each test and alpha level, the proportion of the 1000 replications resulting in a significant statistic provides an estimate of the actual alpha level. To identify those conditions for which a

test did not preserve its level, a criterion interval of $\alpha \pm 2\sqrt{\alpha(1-\alpha)/1000}$ was used for nominal alpha levels of .01 and .05. When $\alpha = .01$ the resulting criterion interval was (.004, .016) and for $\alpha = .05$ the interval was (.036, .063).

Both the parametric and rank analysis of covariance procedures yielded estimated actual alpha levels that were not contained in the criterion intervals given above. Table 4 summarizes those conditions where the estimated actual alpha levels for the parametric analysis of covariance procedure were unacceptable, and Table 5 summarizes those conditions where the estimated actual alpha levels for the rank analysis of covariance procedure were unacceptable. For each condition identified, the estimated actual alpha levels are provided. Since conditions involving non-normal conditional distributions were replicated twice, two estimated actual alpha levels are provided for each of the conditions identified in Tables 4 and 5.

The results in Table 4 suggest that while the parametric analysis of covariance procedure did have estimated actual alpha levels that were unacceptable, relative to the pre-established criterion interval, those aberrant alpha levels were never too distant from the criterion interval limits. In addition, for the cases where 2 replicates were simulated, the estimated alpha levels were never outside of the criterion interval in

Table 4

Conditions Which Resulted in an Actual Probability of a Type I Error Outside the Criterion Level: Parametric ANCOVA

Skew	Size of Conditional Variance	Sample Size	Ratio of Variances	Nominal Alpha .01	.05
+.25	$1-.3^2$	20	1	.020,.011[a]	
+.25	$1-.3^2$	20	2	.014,.011	
+.25	$1-.7^2$	40	1		.068,.051
+.75	$1-.3^2$	40	1		.054,.065
-.25/.50	$1-.7^2$	40	1	.018,.004	
.25/.75	$1-.3^2$	20	1	.017,.010	
.25/.75	$1-.3^2$	40	1	.017,.011	
.25/.75	$1-.7^2$	40	1	.003,.009	

Note: For all conditions identified in this table only one of the two replications of the condition resulted in a Type I error probability outside the criterion interval.

a. estimated actual alpha levels.

both replications. These findings suggest that when the assumptions of conditional normality and/or homogeneity of conditional variances are violated, to the degree examined in this study, the parametric analysis of covariance procedure appears to have preserved its level.

The results provided in Table 5 indicate that the rank analysis of covariance procedure yielded estimated actual alpha levels well above the upper criterion interval limit in both replications of each nominal alpha level for several of the experimental conditions simulated. When the conditional distributions had the same degree of skew ($\pm$ .75), but were skewed in opposite directions, and the size of the conditional variance was $1-.3^2$, aberrant alpha levels were observed for all combinations of sample size and ratio of variances. Unacceptable alpha levels were also observed for these conditional distributions (skew $\pm$ .75) when the size of conditional variance was $1-.7^2$, sample size was 40, and the ratio of variances was 2. For all other experimental conditions identified in Table 5, the estimated actual alpha levels were above the upper limit of the criterion interval for only one of the replicates and these alpha levels tended to be fairly close to the upper limit. It appears then, that the levels of the parametric and rank analysis of

Table 5

Conditions Which Resulted in an Actual Probability of a Type I Error Outside the Criterion Level: Rank ANCOVA

Skew	Size of Conditional Variance	Sample Size	Ratio of Variances	Nominal Alpha .01	Nominal Alpha .05
Normal	$1-.7^2$	40	1		.070[c]
+ .75	$1-.3^2$	20	1	.022,.024[a]	.079,.078[a]
∓ .75	$1-.3^2$	20	2	.023,.021[a]	.075,.069[a]
∓ .75	$1-.7^2$	20	2		.063,.066[b]
∓ .75	$1-.3^2$	40	1	.022,.040[a]	.086,.095[a]
∓ .75	$1-.7^2$	40	1	.019,.016[b]	.048,.066[b]
∓ .75	$1-.7^2$	40	2	.018,.020[a]	.074,.075[a]
.25/.50	$1-.3^2$	40	1	.017,.011[b]	
.25/.50	$1-.7^2$	40	2	.017,.010[b]	
.25/.75	$1-.3^2$	40	1	.022,.015[b]	
.25/.75	$1-.7^2$	40	2		.063,.064[b]

a. in both replications of this condition the actual probability of a Type I error was outside the criterion interval.
b. in one of the two replications of this condition the actual probability of a Type I error was outside the criterion interval.
c. there was only one replicate of this condition.

covariance procedures are preserved for the vast majority of conditions simulated in this study. The exceptions occurred when the rank analysis of covariance was employed with conditional distributions that were moderately skewed in opposite directions and the size of the conditional variance was large ($1-.3^2$), or with conditional distributions that were moderately skewed in opposite directions and the groups (sample size 40) had unequal conditional variances and the size of the conditional variance was small ($1-.7^2$). These findings are comparable to those obtained in other research studies which examined empirically, other more extreme violations of the conditional normality and/or homogeneity of conditional variance assumptions (Abunnaja, 1983; Conover and Iman, 1980; Olejnik and Algina, 1983; Olejnik, Algina and Abdel-Fattah, 1984). Finally, while an investigation of the actual power of the rank analysis of covariance to detect differences in skew and/or variance was not a major objective of this study, the results given above would also suggest that for the levels of skew investigated, the rank procedure has little power to detect differentially skewed conditional distributions that do not differ in conditional means.

Power Differences

A second, and perhaps most important purpose of the study, was to compare the statistical powers of the rank and parametric analysis of covariance procedures in two research situations. The first situation considered was one in which the researcher was interested in detecting differences in conditional means, and the conditional distributions encountered in the study differed in skew and/or variance across groups. In the second, the researcher was interested in detecting differences in conditional distributions and expected that the treatments would have resulted in conditional distributions that differed in mean and skew and/or variance. For each experimental condition investigated, observations were simulated from two populations which had conditional mean functions that differed by a factor of .20, .40, .60, or .80 marginal standard deviation units, when sample sizes were 20 and 20. When sample sizes were 40 and 40, the conditional mean functions differed by a factor of .15, .30, .45, or .60 marginal standard deviation units. The 1000 replications for each condition were analyzed using the parametric and rank analysis of covariance procedures and nominal alpha levels of .01 and .05. For each test and nominal alpha level, the proportion of the 1000 replications resulting in a significant statistic provided an estimate of the statistical power of the test.

The results are reported in a series of tables that, for clarity, have been further divided into two sections. In the first section, reported results pertain to experimental conditions where the assumption of conditional normality was the only assumption violated. In the second section, the reported results reflect the effects of a combination of conditional non-normality and between group heterogeneity of variance. Each table reports the power of the parametric analysis of covariance procedure (P_p) and the power differences between the parametric and rank analysis of covariance procedures (P_p-P_r) for nominal $\alpha = .05$. The results for nominal $\alpha = .01$ were quite similar. For each condition in each table, four parametric power estimates and estimated power differences are provided, one for each of the four effect sizes investigated for that condition.

Homogeneous Conditional Variances

Case A. Table 6 provides the parametric power estimates (P_p) and the power differences (P_p-P_r) for the normal conditional distributions and for the experimental conditions where the conditional distributions differed in skew and there was a positive correlation between the sign of skew and C_j from (2). The most obvious characteristic of the results was that, with one exception, the estimated power difference were

Table 6

Power of the Parametric ANCOVA (P_p) and Power Difference (P_p-P_r) between the Parametric and Rank ANCOVA

Sample Size	P	Effect Size	Conditional Distributions: Normal P_p	Normal P_p-P_r	$\pm$ .25[a] P_p	$\pm$ .25[a] P_p-P_r	$\pm$ .75[a] P_p	$\pm$ .75[a] P_p-P_r
20,20	.3	.20	104[b]	-001	129	021	115	067
		.40	260	023	262	040	205	085
		.60	492	027	500	061	456	131
		.80	742	027	750	073	728	176
	.7	.20	158	019	144	027	101	021
		.40	393	037	407	056	358	090
		.60	732	064	734	061	751	144
		.80	929	029	927	029	957	053
40,40	.3	.15	102	001	100	011	095	050
		.30	300	017	274	062	280	158
		.45	565	022	544	086	550	232
		.60	780	022	821	101	814	235
	.7	.15	161	004	140	017	086	060
		.30	450	029	466	056	310	123
		.45	776	044	807	069	664	151
		.60	960	024	967	008	913	050

a. all results refer to the conditions with a positive correlation between sign of skew and C_j.
b. decimal points are omitted.

positive indicating that the parametric analysis of covariance was a more powerful test for both the normal and the non-normal conditional distributions. Since, with the one exception, the power differences were positive, the effects of the factors manipulated in the study have little implication for the choice of a statistical test. Nevertheless, it may be of interest to note that for a particular sample size, size of conditional variance (here strength of covariate-dependent variable correlation), and effect size the power advantages favoring the parametric analysis of covariance actually increase as the conditional distributions become more non-normal. Consider the most extreme case, where sample size was 40 and $\rho = .3$ the largest power difference for the normal conditional distributions was .02 while the largest power difference for the conditional distributions having degree of skew $\pm$ .75 was .235. While less extreme changes occur for the other combinations of sample size and ρ, substantial changes do occur for each combination. It is also of interest to note that, in general, for a particular combination of sample size, covariate-dependent variable correlation, and effect size the power of the parametric analysis of covariance is affected little by changes in the shape of the conditional distributions. The effect of shape of the conditional distributions on

power differences occurs because the power of the rank analysis of covariance tends to decrease as the conditional distributions become increasingly non-normal.

Table 7 gives the parametric power (P_p) and the power differences (P_p-P_r) for the experimental conditions where the conditional distributions differed in sign of skew and there was a negative correlation between sign of skew and C_j from (2). The power differences were inconsequential when the conditional distributions had degrees of skew $\pm$ .25. However, when conditional distributions had degrees of skew $\pm$ .75 some reasonably important power differences favoring the rank analysis of covariance procedure occurred when sample size was 20 or 40 and ρ was .3 or when sample size was 40 and ρ was .7. The largest differences occurred when sample size was 40 and ρ was .3. Differences of this magnitude certainly have implication for the choice of a statistical procedure.

In interpreting these power differences though, it must be recalled that when the rank analysis of covariance was used to test the equality of conditional means, the level of the test was not preserved when ρ was .3 and sample size was 20 or 40 (see Table 3). If the purpose of a particular analysis is to test the equality of conditional means the rank analysis

Table 7

Power of the Parametric ANCOVA (P_p) and Power Difference (P_p-P_r) between the Parametric and Rank ANCOVA: Case A

Sample Size		Effect Size	Conditional Distributions ± .25[a] P_p	± .25[a] P_p-P_r	± .75[a] P_p	± .75[a] P_p-P_r
20,20	.3	.20	133	002	120	-049
		.40	214	-001	281	-040
		.60	491	006	492	-043
		.80	715	027	682	-033
	.7	.20	137	000	149	-031
		.40	395	009	408	008
		.60	705	026	693	-007
		.80	966	026	903	-017
40,40	.3	.15	101	-019	143	-085
		.30	277	-027	297	-119
		.45	545	-001	547	-102
		.60	768	-017	767	-069
	.7	.15	183	-013	137	-043
		.30	486	017	421	-046
		.45	873	041	740	-025
		.60	948	009	936	002

Note: All results refer to the conditions with a negative correlation between sign of skew and C_j.

a. decimal points are omitted.

procedure could not be recommended for these conditions. It would however be the preferred procedure when ρ is large ($\rho = .7$) and sample size is large ($n = 40$) since the test did preserve its level in this condition and enjoys a small power advantage over the parametric procedure. For the remaining experimental conditions the choice between the two procedures appears to be relatively inconsequential. The levels of both tests were preserved when they were used to test the equality of conditional distributions. The rank analysis of covariance could therefore be recommended in this situation when ρ is small ($\rho = .3$) and sample size is either 20 or 40 or when ρ is large ($\rho = .7$) and sample size is 40. For all other experimental conditions, the choice between statistical procedures would again be relatively inconsequential. The results do suggest, however, that a choice may not be inconsequential with larger sample sizes. When conditional distributions were skewed $\pm .25$ and the covariate-dependent variable correlation was small ($\rho = .3$) the parametric analysis of covariance enjoyed power advantages for a sample size of 20. When sample size was increased to 40, though, the rank analysis of covariance procedure enjoyed power advantages of approximately the same magnitude. When the conditional distributions were skewed $\pm .75$ and the covariate-dependent variable

correlation was weak ($\rho = .3$) the rank analysis of covariance enjoyed power advantages for both sample sizes, but those power advantages increased with the increase in sample size. Taken together these results would suggest that when the covariate-dependent variable correlation is weak and sample sizes are large ($n > 40$) the rank analysis of covariance procedure may be preferred even with conditional distributions slightly skewed in opposite directions.

Case B. Table 8 gives the parametric power (P_p) and the power differences ($P_p - P_r$) for those experimental conditions where conditional distributions differed in degree of skew and there was a positive correlation between degree of skew and C_j from (2). Here, all power differences are positive suggesting that the parametric analysis of covariance procedure is more powerful than the rank analysis of covariance for all combinations of sample size, strength of covariate-dependent variable correlation, and magnitude of difference in degree of skew. Thus, when conditional distributions differ in degree of skew and there is a positive correlation between degree of skew and C_j from (2), the parametric analysis of covariance procedure would be preferred.

Table 8

Power of the Parametric ANCOVA (P_p) and Power Differences (P_p-P_r) between the Parametric and Rank ANCOVA: Case B

Sample Size		Effect Size	Conditional Distributions .25, .50 P_p	P_p-P_r	.25, .75 P_p	P_p-P_r
20,20	.3	.20	088	007	106	015
		.40	242	031	245	029
		.60	495	040	490	040
		.80	753	058	715	048
	.7	.20	124	006	140	014
		.40	406	050	415	066
		.60	727	066	723	062
		.80	924	037	932	036
40,40	.3	.15	097	007	109	017
		.30	268	040	298	076
		.45	543	057	542	073
		.60	787	052	822	081
	.7	.15	154	026	149	021
		.30	461	059	449	068
		.45	803	041	791	069
		.60	968	021	966	039

a. all results refer to the conditions with a positive correlation between C_i and degree of skew.
b. decimal points are omitted.

Table 9 gives the parametric powers (P_p) and power differences (P_p-P_r) for the experimental conditions where the conditional distributions differed in degree of skew and there was a negative correlation between degree of skew and C_j from (2). In general, the power differences are of little consequence. However, the parametric analysis of covariance enjoyed a small power advantage for both levels of difference in skew when the covariate-dependent variable correlation was large ($\rho = .7$) and sample size was 20. The rank analysis of covariance enjoyed a power advantage of about the same magnitude when the conditional distributions had degrees of skew .25 and .75, and the covariate-dependent variable correlation was weak ($\rho = .3$) and sample size was 40. It is also of interest to note that for a given level of difference in degree of skewness and strength of covariate-dependent variable correlation, an increase in sample size had an obvious effect on power difference. As sample size increased, any power differences in favor of the parametric analysis of covariance tend to decline and the power differences in favor of the rank analysis of covariance tend to increase. This would suggest that with even larger sample sizes, the rank analysis of covariance may enjoy practically important power advantages for a number of experimental conditions involving various combinations of strength of covariate-dependent variable correlation and difference in degree of skew. The work of

Table 9

Power of the Parametric ANCOVA (P_p) and Power Differences (P_p-P_r) between the Parametric and Rank ANCOVA: Case B

Sample Size		Effect Size	Conditional Distributions .25, .50 P_p	.25, .50 P_p-P_r	.25, .75 P_p	.25, .75 P_p-P_r
20,20	.3	.20	109	-003	107	-018
		.40	277	025	260	-013
		.60	486	010	475	-024
		.80	710	006	734	-012
	.7	.20	136	-002	146	005
		.40	415	051	382	061
		.60	696	040	733	047
		.80	908	035	915	033
40,40	.3	.15	103	-008	122	-026
		.30	275	-013	289	-059
		.45	524	000	542	-042
		.60	774	004	755	-036
	.7	.15	157	005	145	-014
		.30	440	009	450	011
		.45	798	031	788	020
		.60	957	005	955	007

a. all results refer to the conditions with a negative correlation between C_i and degree of skew.
b. decimal points are omitted.

Olejnik and Algina (1984) would suggest, however, that the greatest power increases might be expected in the middle of the power curves where the parametric power ranged from .4 to .6.

Heterogeneous Conditional Variances

A number of experimental conditions were simulated in this study so that the combined effects of conditional non-normality and between group heterogeneity of variance could be examined. The results obtained from the simulation of these experimental conditions were quite similar to those obtained for experimental conditions where conditional non-normality was the only violation considered, and detailed interpretation of the results obtained in the heterogeneous case would therefore be redundant. For this reason, a very general discussion of the results is given.

Case A. Table 10 provides the parametric power estimates (P_p) and the power differences (P_p-P_r) for the normal conditional distributions and for the experimental conditions where the conditional distributions differed in sign of skew and there was a positive correlation between sign of skew and C_j from equation (2). The power differences are all positive suggesting that the parametric analysis of covariance was again a more powerful test for both the normal and non-normal conditional distributions. Also, for a particular sample

Table 10

Power of the Parametric ANCOVA (P_p) and Power Difference ($P_p - P_r$) between the Parametric and Rank ANCOVA

Sample Size	Size of Conditional Variance	Effect Size	Conditional Distributions: Normal P_p	Normal $P_p - P_r$	$\pm$ .25 P_p	$\pm$ .25 $P_p - P_r$	$\pm$ .75 P_p	$\pm$ .75 $P_p - P_r$
20,20	$1-.3^2$	.20	095[a]	005	078	001	073	018
		.40	236	004	214	031	216	072
		.60	460	027	459	060	480	135
		.80	716	054	710	054	737	136
20,20	$1-.7^2$	.20	144	018	148	019	110	004
		.40	370	034	419	062	391	097
		.60	724	051	754	080	734	111
		.80	931	035	939	039	952	050
40,40	$1-.7^2$	.15	138	022	137	021	133	040
		.30	475	048	433	075	424	121
		.45	798	030	779	079	786	132
		.60	951	015	956	029	973	060

Note: All results refer to heterogeneous variance conditions where there was a positive correlation between sign of skew and C_i.

a. decimal points are omitted.

size, size of conditional variance, and effect size the power advantages favoring the parametric analysis of covariance increase as the conditional distributions become more non-normal.

Table 11 gives the parametric powers (P_p) and the power differences (P_p-P_r) for the experimental conditions where the conditional distributions differed in sign of skew and there was a negative correlation between sign of skew and C_j from (2). Again, power differences appear to have been inconsequential when the conditional distributions had degrees of skew $\pm$ .25. When degree of skew was $\pm$ .75, however, some reasonably important power differences favoring the rank analysis of covariance procedure were observed. As with the homogeneous variance conditions, the greatest power advantages in favor of the rank analysis procedure occurred when the size of the conditional variance was large ($1-.3^2$).

When interpreting these power differences it must be recalled that when the rank analysis of covariance is used to test the equality of conditional means, aberrant alpha levels were observed when the size of the conditional variance was large ($1-.3^2$) and sample size was 20 or 40, or when the size of the conditional variance was small ($1-.7^2$) and sample size was 40 (see Table 5). If the purpose of analysis is to test the equality of conditional means, the rank analysis of covariance could not be

Table 11

Power of the Parametric ANCOVA (P_p) and Power Difference (P_p-P_r) between the Parametric and Rank ANCOVA: Case A

Sample Size	Size of Conditional Variance	Effect Size	Conditional Distributions ± .25 P_p	± .25 P_p-P_r	± .75 P_p	± .75 P_p-P_r
20,20	$1-.3^2$	.20	098[a]	000	119	-039
		.40	236	002	301	-044
		.60	449	008	504	-044
		.80	701	012	712	-007
20,20	$1-.7^2$	.20	151	-007	175	-025
		.40	412	016	410	-010
		.60	728	036	690	019
		.80	930	051	899	041
40,40	$1-.3^2$	.15				
		.30				
		.45				
		.60				
40,40	$1-.7^2$	.15	177	017	174	-032
		.30	463	001	472	-041
		.45	780	033	772	-010
		.60	954	016	935	004

Note: All results refer to the heterogeneous variance conditions with a negative correlation between sign of skew and C_j.

a. decimal points are omitted.

recommended for these conditions. For the remaining experimental conditions a choice between the two procedures would be inconsequential. When testing the equality of conditional distributions, the levels of both tests were preserved, thus the rank analysis of covariance may be a more powerful procedure when the size of the conditional variance is large ($1-.3^2$) and the conditional distributions are moderately non-normal (skew $\pm$ .75).

Case B. In this case, there were eight possible patterns of positive and/or negative correlations among C_j, S_j, and degree of skew. Since data were simulated for just two groups, once the correlations between two pairs of variables was specified, the third correlation was automatically determined. This meant that there were four distinct patterns of correlation among the three variables. The relationship used to limit the number of simulations in Case A, $Y^* = -Y$, was not applicable for experimental conditions in Case B. As a result all four patterns of correlations among C_j, S_j, and degree of skew were simulated. In Tables 12 through 15 the parametric power estimates (P_p) and estimated power differences (P_p-P_r) are reported for the experimental conditions where the conditional distributions were differentially skewed, in the same direction. Each table provides results for experimental conditions for one of

Table 12

Power of the Parametric ANCOVA (P_p) and Power Difference (P_p-P_r) between the Parametric and Rank ANCOVA: Case B

Sample Size	Size of Conditional Variance	Effect Size	Conditional Distributions .25, .50 P_p	.25, .50 P_p-P_r	.25, .75 P_p	.25, .75 P_p-P_r
20,20	$1-.3^2$	.20	097[a]	012	091	025
		.40	229	032	215	050
		.60	483	075	434	103
		.80	724	071	700	102
20,20	$1-.7^2$	.20	134	016	128	020
		.40	384	064	407	076
		.60	727	073	754	075
		.80	939	048	945	042
40,40	$1-.3^2$	.15				
		.30				
		.45				
		.60				
40,40	$1-.7^2$	.15	146	033	129	039
		.30	443	086	421	098
		.45	795	082	785	095
		.60	959	028	958	027

Note: All results refer to heterogeneous variance conditions with a positive correlation between degree of skew and C_i and a positive correlation between degree of skew and S_i.

a. decimal points are omitted.

Table 13

Power of the Parametric ANCOVA (P_p) and Power Difference (P_p-P_r) between the Parametric and Rank ANCOVA: Case B

Sample Size	Size of Conditional Variance	Effect Size	Conditional Distributions .25, .50 P_p	.25, .50 P_p-P_r	.25, .75 P_p	.25, .75 P_p-P_r
20,20	$1-.3^2$	.20	104[a]	008	099	002
		.40	250	016	246	001
		.60	506	038	502	027
		.80	722	037	744	049
20,20	$1-.7^2$	.20	158	009	150	-007
		.40	406	030	396	008
		.60	731	050	706	018
		.80	917	023	925	021
40,40	$1-.3^2$	.15				
		.30				
		.45				
		.60				
40,40	$1-.7^2$	.15	157	010	170	016
		.30	464	009	450	041
		.45	795	034	780	031
		.60	957	014	953	041

Note: All results refer to the heterogeneous variance conditions with a positive correlation between degree of skew and C_j and a negative correlation between degree of skew and S_j^2

a. decimal points are omitted.

Table 14

Power of the Parametric ANCOVA (P_p) and Power Difference ($P_p - P_r$) between the Parametric and Rank ANCOVA: Case B

Sample Size	Size of Conditional Variance	Effect Size	Conditional Distributions .25, .50 P_p	.25, .50 $P_p - P_r$	.25, .75 P_p	.25, .75 $P_p - P_r$
20,20	$1-.3^2$	.20	112[a]	-007	110	-034
		.40	235	-021	268	-044
		.60	472	015	494	-021
		.80	708	011	711	-005
20,20	$1-.7^2$	.20	141	001	120	-007
		.40	367	030	375	010
		.60	696	041	674	022
		.80	891	020	910	035
40,40	$1-.3^2$	.15				
		.30				
		.45				
		.60				
40,40	$1-.7^2$	.15	166	-014	149	026
		.30	452	006	464	-015
		.45	773	027	768	-007
		.60	955	026	945	011

Note: All results refer to heterogeneous variance conditions with a negative correlation between degree of skew and C_i and a positive correlation between degree of skew and S_i.

a. decimal points are omitted.

Table 15

Power of the Parametric ANCOVA (P_p) and Power Difference (P_p-P_r) between the Parametric and Rank ANCOVA: Case B

Sample Size	Size of Conditional Variance	Effect Size	Conditional Distributions .25, .50 P_p	.25, .50 P_p-P_r	.25, .75 P_p	.25, .75 P_p-P_r
20,20	$1-.3^2$	.20	105[a]	013	089	-015
		.40	264	025	239	022
		.60	510	054	478	041
		.80	721	040	719	030
20,20	$1-.7^2$	.20	142	014	129	011
		.40	398	023	411	042
		.60	727	055	715	044
		.80	930	052	926	048
40,40	$1-.3^2$	.15				
		.30				
		.45				
		.60				
40,40	$1-.7^2$	.15	148	012	152	011
		.30	441	022	440	042
		.45	804	063	760	050
		.60	964	018	948	023

Note: All results refer to heterogeneous variance conditions with a negative correlation between degree of skew and C_i and a negative correlation between degree of skew and S_i.

a. decimal points are omitted.

the four possible patterns among C_j, S_j, and degree of skew. Careful review of the entries in these tables would indicate that, with few exceptions, the power differences are positive suggesting that the parametric analysis of covariance procedure was a more powerful test. The greatest power differences favoring the parametric procedure were obtained for experimental conditions where there was a positive correlation between degree of skew and C_j and a positive correlation between degree of skew and S_j (Table 12). The rank analysis of covariance enjoyed small power advantages in experimental conditions where the correlation between degree of skew and C_j was negative, the correlation between degree of skew and S_j, was positive, the conditional distributions had degrees of skewness .25, .75, the size of the conditional variance was large ($1-.3^2$) and the sample size was 20. (See Table 14) For all other experimental conditions, the choice of a statistical procedure would be inconsequential. However, the preponderance of power differences favored the parametric procedure suggesting that in the heterogeneous case, when conditional distributions are differentially skewed, a general use of the parametric procedure is indicated.

CHAPTER V

CONCLUSIONS

An important consideration for the researcher who must choose between a parametric or nonparametric statistical test is the effect that violations of assumptions, separately and in combination, have on type I errors and statistical power for the alternative procedures. Specific information of this kind for a variety of combinations and degrees of violations of assumptions could be most useful to the researcher. Armed with this information, astute decisions regarding the appropriateness of a statistical procedure may be made. In the present study a limited number of discrepancies between expected and estimated actual alpha levels were observed for a large number of experimental conditions where the assumptions of conditional normality and/or between group homogeneity of variance were violated to varying degrees. Those aberrant alpha levels have been reported in Tables 4 and 5. Further, comparisons of the statistical powers of the parametric and rank analysis of covariance procedures were made for those same experimental conditions. The results of those

comparisons have been highlighted in Tables 6 through 15. While detailed discussions followed each of the Tables 4 through 15, certain conclusions follow. These conclusions may provide guidelines for researchers who must choose between the parametric and rank analysis of covariance procedures.

Conclusion 1. The level of the parametric analysis of covariance is preserved for moderate violations of the conditional normality and/or between group homogeneity of variance assumptions. As evidenced in Table 4, when sample sizes are equal, conditional non-normality and/or between group heterogeneity of variance produced few discrepancies between expected alpha levels and estimated actual alpha levels. Further, even when aberrant alpha levels were observed for a particular experimental condition those aberrant levels were not replicated. Thus, when faced with only moderate violations of the conditional normality and/or between group homogeneity of variance assumptions the researcher need not dispose of the parametric analysis of covariance in favor of some alternative distribution - free procedure.

Conclusion 2. The level of the rank analysis of covariance is,with few exceptions, preserved when conditional distributions are moderately non-normal and conditional variances across groups are unequal (ie ratio of 2). The exceptions were identified in Table 5. In

general though they indicate that the rank analysis of covariance may be quite sensitive to moderate differences in direction of skew when the size of the conditional variance is large. The rank analysis of covariance may therefore be an inappropriate test of a given hypothesis when conditional distributions differ in this way.

Conclusion 3. When testing the null hypothesis of equality of conditional means, the parametric analysis of covariance is the procedure of choice. Tables 6 through 15 indicate that the parametric analysis of covariance, by and large, enjoys power advantages for most experimental conditions examined in this study. When the rank analysis of covariance had any power advantages they often occurred for experimental conditions where the level of the rank procedure was not preserved for this particular hypothesis test. In short very little power will be lost and possibly much power gained by the more general use of the parametric procedure to test the equality of conditional means for the degrees of violations examined in this study.

Conclusion 4. When testing the null hypothesis of equality of conditional distributions, and there is a positive correlation between sign or degree of skew and height of the conditional mean function, the parametric analysis of covariance is consistently more powerful and

is the preferred procedure. The power differences reported in Tables 6, 8, 10, 12, and 13 (conditions where there was a positive correlation between sign or degree of skew and C_j) are, with very few exceptions, positive indicating greater power for the parametric analysis of covariance. This would suggest that there would be little reason to use the rank analysis procedure to test the hypothesis of equality of conditional means or equality of conditional distributions. In fact, a large number of the power differences found in these tables are quite large suggesting practically important power advantages in favor of the parametric analysis of covariance.

Conclusion 5. When testing the null hypothesis of equality of conditional distributions, and there is a negative correlation between sign or degree of skew and height of conditional mean function, the choice of a statistical procedure is often inconsequential with the parametric procedure enjoying power advantages for some conditions and the rank procedure enjoying comparable power advantages for other dissimilar conditions. Careful review of the power differences in Tables 7, 9, 11, 14 and 15 does however lead one to the conclusion that the rank analysis of covariance enjoys power advantage for a limited number of experimental conditions. Those conditions often involved conditional distributions that were moderately

non-normal ($\pm$ .75 or .25, .75) having a large conditional variance ($1-.3^2$). The parametric analysis of covariance, on the other hand, enjoyed power advantages for a greater number of experimental conditions where the conditional distributions were mildly to moderately non-normal and the size of the conditional variance was small ($1-.7^2$). This would suggest that if a reasonable covariate has been selected (ie. one resulting in a large covariate-dependent variable correlation) the parametric procedure may be preferred even for those conditions where there is a negative correlation between sign or degree of skew and height of the conditional mean function.

Limitations. There were obvious limitations in the present study that restrict its generalizability. First, the present study only considered slight to moderate violations of the conditional normality and between group homogeneity of variance assumptions. More extreme violations of the assumptions could result in very different conclusions regarding the choice of a statistical procedure. However, the aim of the present study was to consider degrees of violations that would be likely in actual practice.

Second, the inability (due to cost considerations) to simulate all of the experimental conditions for a sample size of 40 was seen as a limitation of the study, although not a severe one. The experimental conditions simulated

suggest certain patterns in the power differences that are rather obvious. While the missing observations therefore appear to be obvious there is no concrete evidence to support this claim.

Finally, only the rank transform alternative to the analysis of covariance was considered in this study. It is true that the rank transform procedure and Quade's and Puri and Sen's procedures have exhibited similar Type I error rates and power in other simulations. However, that finding does not rule out the possibility that these three procedures may yield different results with the kind of data generated in this study. It seems then that a comparison of the nonparametric procedures applied to data of the kind generated here, would be most useful.

Given these limitations, the results of this study should provide useful information for the researcher who must choose between the parametric and rank analysis of covariance procedures in the face of modest degrees of conditional non-normality and between group heterogeneity of variances.

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BIOGRAPHICAL SKETCH

Samuel L. Seaman was born 12 November, 1954, in Brookhaven, Pennsylvania. He graduated from DeLand Senior High School in June, 1972. In May, 1976, he received the Bachelor of Arts degree from Stetson University. He taught school for three years and in 1979 returned to Stetson University to do graduate work, where he received the Master of Education degree in May, 1980. In August, 1981 he began work toward the Ph.D. degree at the University of Florida.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

James J. Algina, Chairman
Associate Professor of
Foundations of Education

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Gordon G. Bechtel
Professor of Marketing

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Linda M. Crocker
Associate Professor of
Foundations of Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Stephen F. Olejnik
Associate Professor of
Foundations of Education

This dissertation was submitted to the Graduate Faculty of the Department of Foundations of Education in the College of Education and to the Graduate School, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1984

Chairman, Foundations of Education

Dean for Graduate Studies and Research